Name:  

Math 106C: Winter 2012  
Quiz 1: January 30

Please write your final answer in the space provided. For full credit you must show your work. Good Luck!

1. Let \( I = \int_{a}^{b} f(x) \, dx \), where \( f \) is positive and concave down over the interval \([a, b]\). Indicate whether, for all \( n \geq 1 \), the statement must be true, cannot be true, or may be true.

   (a) \( R_n \leq I \)  
   (b) \( T_n \leq I \)

   \( 1a \) may be  
   \( 1b \) must be

2. The graph below depicts the velocity (in mph) of a bike over a period of 8 hours. The distance traveled by the car in those 8 hours can be calculated by finding the area under the curve. Use the Midpoint Rule with 4 intervals (i.e., \( n = 4 \)) to estimate the distance traveled.

\[ \Delta x = \frac{8-0}{4} = 2 \]

\[ v(m_1) \approx 7.5, \quad v(m_2) \approx 8 \]

\[ v(m_3) \approx 5, \quad v(m_4) \approx 18.5 \]

\[ M_4 = 2(7.5 + 8 + 5 + 18.5) = 78 \]

\[ \text{78 hours} \]

OVER
3. Recall the formulas:
\[
|I - L_n| \leq \frac{K_1(b - a)^2}{2n} \\
|I - R_n| \leq \frac{K_1(b - a)^2}{2n} \\
|I - T_n| \leq \frac{K_2(b - a)^3}{12n^2} \\
|I - M_n| \leq \frac{K_2(b - a)^3}{24n^2}
\]

Let \( I = \int_{-1}^{2} f(x) \, dx \). Below are two graphs. The one on the left is a graph of \( f'(x) \). The one on the right is a graph of \( f''(x) \).

Find \( a \) and \( b \).

What is the smallest value of \( n \) that guarantees \( |I - L_n| \leq 0.0012 \)?

(3) \( 7875 \)

\(|f'(x)| \leq 1.75 \) for \( x \in [-1, 2] \)

So take \( K_1 = 1.75 \) (\( K_1 = 2 \) is also ok, b/c 2 is above the circled peak. It would be wrong to take \( K_1 = 1.5 \), b/c the accuracy of the graph doesn't guarantee \(|f'(x)| \leq 1.5 \) for \( x \in [-1, 2] \)).

\[
\frac{1.75(2-1)^2}{2n} \leq 0.001
\]

\[
\frac{1.75(3)^2}{2(0.001)} \leq n
\]

\( 7875 \leq n \) so can take \( n = 7875 \)