NAME:

Show ALL your work CAREFULLY.

(a) Consider the initial value problem

\[ \frac{dy}{dx} = x - y \]

with \( y(1) = 2 \).

Estimate the value \( y(2) \) (when \( x = 2 \)) of the solution using Euler’s method with two steps with initial point \((1, 2)\). DO THIS BY HAND and show all your steps.

The initial point is \((t_0, y_0) = (1, 2)\). The interval \([1, 2]\) is subdivided into two equal subintervals so that \( \Delta x = 0.5 \). Now, \( y_1 = y_0 + f(t_0, y_0) \cdot \Delta x = \frac{2}{1} = 1.5 \). Thus, \( y_2 = y_1 + f(t_1, y_1) \cdot \Delta x = 1.5 + \frac{(1 - 2)(0.5)}{1} = 1.5 + 0 = 1.5 \).

(b) Find the exact area of the region bounded by the curve \( y = x^3 + 1 \), the line \( y = 2 \), and the \( y \)-axis.

The region in question is given by the following figure.

Using vertical slices, the area of the bounded region is given by

\[ \int_0^1 2 - (x^3 + 1) \, dx = \int_0^1 1 - x^3 \, dx = \left. x - \frac{x^4}{4} \right|_0^1 = \frac{3}{4} \]

(c) Set up (but DO NOT evaluate) a definite integral representing the exact volume of the solid obtained by rotating the region in (b) around the \( x \)-axis.

The volume of the solid is given by

\[ \int_0^1 \pi(2)^2 - \pi(x^3 + 1)^2 \, dx = \pi \int_0^1 3 - 2x^3 - x^6 \, dx. \]