Let $A$ be the region in the first quadrant bounded by the curves $y = \sqrt{1 - x^2}, y = \sqrt{1 - \frac{x}{4}}$ and the line $y = 0$.

(a) Set up (do not evaluate) a definite integral representing the area of the region $A$. [Hint: Sketch the region $A$ first.]

Using horizontal slices, the length of a typical slice (in green) is the difference between the $x$-coordinates of the right and the left endpoints, i.e., $4(1 - y^2) - \sqrt{1 - y^2}$. It follows that the area of the region $A$ is given by

$$\int_0^1 4(1 - y^2) - \sqrt{1 - y^2} \, dy.$$ 

(b) Find the exact volume of the solid formed by rotating the region $A$ about the $y$-axis.

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Using horizontal slices again, a typical slice of this solid is a “washer”. It follows that the volume of the solid is given by

\[ V = \int_0^1 \left[ \pi (4(1-y^2))^2 - \pi (\sqrt{1-y^2})^2 \right] dy \]

\[ = \pi \int_0^1 16(1-2y^2+y^4) - (1-y^2) \, dy \]

\[ = \pi \left. \left( 15 - 31y^2 + 16y^4 \right) \right|_0^1 = \pi \left( 15 - \frac{31}{3} + 16 \right) = \frac{118\pi}{15}. \]