Math 106B - Quiz 3 - January 29, 2007

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

1. (10 pts.) Consider the initial value problem

\[ \frac{dy}{dt} = 2y + t, \quad y(2) = 3. \]

Use Euler's method with two steps of size \( \Delta t = 0.5 \) to estimate \( y(3) \).

\( \Delta t = \frac{1}{2} \). When \( t = 2 \), \( y = 3 \).

First pt: \( t_0 = 2, \quad y_0 = 3 \). \[ \frac{dy}{dt} = 2(3) + 2 = 8. \] \[ dy = \frac{dy}{dt} \Delta t = 8 \cdot \frac{1}{2} = 4 \]

Second pt: \( t_1 = t_0 + \Delta t = 2 + \frac{1}{2} = 2.5 \)

\[ y_1 = y_0 + dy = 3 + 4 = 7 \]

Third pt: \( t_2 = t_1 + \Delta t = 2.5 + \frac{1}{2} = 3 \)

\[ y_2 = y_1 + dy = 7 + 8 \cdot \frac{1}{2} = 15.25 \]

So, when \( t = 3 \), \( y = 15.25 \).

Hence, by Euler's method, \( y(3) \approx 15.25 \). 
(So \( y(3) \approx 15.25 \).)
2. (10 pts.) Consider the region in the $xy$-plane bounded by the graphs of $y = x^2 - 3x + 3$ and $y = -x + 3$. Find the area of this region.

Integrate over the region.

\[ y = x^2 - 3x + 3, \quad y = -x + 3 \]

\[ x^2 - 2x + 3 = -x + 3 \]

\[ x^2 - 2x = 0 \]

\[ x(x-2) = 0 \]

\[ x = 0 \] or \[ x = 2 \]

\[ y = 2 \] at \[ x = 2 \]

\[ y = 3 \] at \[ x = 0 \]

Points \( (0,3) \) and \( (2,1) \)

\[ \text{Area} = \int_{0}^{2} (x^2 - 3x + 3) - (-x + 3) \, dx \]

\[ = \int_{0}^{2} x^2 - x + 3 - x + 3 \, dx \]

\[ = \int_{0}^{2} x^2 - 2x + 6 \, dx \]

\[ = \left[ \frac{x^3}{3} - x^2 + 6x \right]_{0}^{2} \]

\[ = \left( \frac{8}{3} - 4 + 12 \right) - \left( 0 - 0 + 0 \right) \]

\[ = \frac{8}{3} + 12 = \frac{44}{3} \]

\[ = \frac{4}{3} \]