NAME:

Show ALL your work CAREFULLY.

Consider the following given data of a function $f(x)$ on the interval $[-1, 2]$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-1$</th>
<th>-.5</th>
<th>0</th>
<th>.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1.1</td>
<td>1.3</td>
<td>1.7</td>
<td>2.5</td>
<td>3.7</td>
<td>5</td>
<td>8</td>
</tr>
</tbody>
</table>

(a) Find $L_6$ (left hand sum). (Here the subscript $n$ indicates that the interval $[-1, 2]$ is to be divided into $n$ equal subintervals.)

Since $n = 6$, $\Delta x = \frac{2 - (-1)}{6} = 0.5$. Now

$$L_6 = f(-1)\Delta x + f(-0.5)\Delta x + f(0)\Delta x + f(0.5)\Delta x + f(1)\Delta x + f(1.5)\Delta x$$

$$= [f(-1) + f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5)] \cdot \Delta x$$

$$= [1.1 + 1.3 + 1.7 + 2.5 + 3.7 + 5](0.5) = 7.65.$$  

(b) Find $R_6$ (right hand sum).

$$R_6 = [f(-0.5) + f(0) + f(0.5) + f(1) + f(1.5) + f(2)] \cdot \Delta x$$

$$= [1.3 + 1.7 + 2.5 + 3.7 + 5 + 2](0.5) = 11.1.$$  

(c) Find $T_6$ (trapezoid)

$$T_6 = \frac{L_6 + R_6}{2} = \frac{7.65 + 11.1}{2} = 9.375.$$  

(d) Find $M_3$ (mid-point)

Now, with 3 subintervals, $\Delta x = 1$. Thus,

$$M_3 = [f(-0.5) + f(0.5) + f(1.5)] \cdot \Delta x = 8.8.$$  

(e) With the given data, is $T_6$ likely to be an under-estimate or an over-estimate for $I = \int_{-1}^{2} f(x) \, dx$? Explain.

The graph of $f$ appears to be concave up. Thus, $T_6$ is likely to be an over-estimate.