Problem 1. (10 points) Approximating \( \int_{0}^{1} \sin \frac{x}{2} \, dx \) in four different ways gives the following sums:

\[
L_{10} = 0.2208 \quad R_{10} = 0.2688 \quad T_{10} = 0.2448 \quad M_{10} = 0.2449
\]

(a) (5 points) What is the greatest amount of error that \( L_{10} \) can commit for this integral? Answer to four significant figures, and explain your work.

(b) (5 points) What is the greatest amount of error that \( M_{10} \) can commit for this integral? Answer to four significant figures, and explain your work.
Problem 2. (10 points) A velocimeter in an experimental car measures the following velocities during a 10-second time trial.

<table>
<thead>
<tr>
<th>t (sec)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>v (m/sec)</td>
<td>0</td>
<td>2.56</td>
<td>9</td>
<td>17.6</td>
<td>27</td>
<td>36</td>
<td>43.6</td>
<td>49</td>
<td>51.8</td>
<td>51.8</td>
<td>49</td>
</tr>
</tbody>
</table>

Using an approximation on 5 subintervals, come up with a conservative estimate (that is, an underestimate) for the total distance the sports car has traveled over these 10 seconds. Explain how you can be sure your answer is an underestimate.

Problem 3. (5 points) Using three steps of Euler’s method, calculate an approximate value of \( y(7) \) if \( y \) is a solution of the initial-value problem

\[
y' = \frac{y}{x + y}, \quad y(1) = -2.
\]

Sketch what you’ve done on the slope field provided.