NAME:
Show ALL your work CAREFULLY.

(a) Recall that the error committed by using the Right Hand Sum $R_n$ is less than or equal to $K_1 \frac{(b-a)^2}{2n}$ where $|f'(x)| \leq K_1$ for some constant $K_1$. Use this result to give an upper bound for $\left| \int_1^3 x^2 \sin(x^2) \, dx - R_8 \right|$. Here, $f(x) = x^2 \sin(x^2)$.

First, we must find an upper bound $K_1$ for $|f'(x)|$ over the interval $[1,3]$. Since $f(x) = x^2 \sin(x^2)$, we have $f'(x) = 2x \sin(x^2) + x^2 \cos(x^2) \cdot 2x$ by the product rule and the chain rule. Thus, $|f'(x)| = 2|x| |\sin(x^2)| + x^2 \cos(x^2)|$. Over the interval $[1,3]$, the largest value for $x$ is 3 and that for $x^2$ is 9. Neither $\sin(x^2)$ nor $\cos(x^2)$ can exceed 1 in absolute value. It follows that $|f'(x)| \leq 6(1 + 9 \cdot 1) = 60$. Using the error bound for the Right Hand Sum with $K_1 = 60$, we conclude that

$$\left| \int_1^3 x^2 \sin(x^2) \, dx - R_8 \right| \leq \frac{60 \cdot (3 - 1)^2}{2 \cdot 8} = 15.$$ 

(b) Consider the following given data of a function $h(x)$ on the interval $[1,9]$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h(x)$</td>
<td>-1</td>
<td>2</td>
<td>-1</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

Find $M_2$ (mid-point) AND $T_4$ (trapezoid). Here the subscript $n$ indicates that the interval $[1,9]$ is to be divided into $n$ equal subintervals.

First, we compute the Left and the Right Hand Sums $L_4$ and $R_4$. Using the given data, we have

$L_4 = h(1) \cdot \Delta x + h(3) \cdot \Delta x + h(5) \cdot \Delta x + h(7) \cdot \Delta x$
$= (-1 + 2 + (-1) + 6) \cdot \Delta x = 12$;

$R_4 = h(3) \cdot \Delta x + h(5) \cdot \Delta x + h(7) \cdot \Delta x + h(9) \cdot \Delta x$
$= (2 + (-1) + 6 + 3) \cdot \Delta x = 20$.

It follows that $T_4 = \frac{L_4 + R_4}{2} = 16$. For $M_2$, note that $\Delta x = 4$ since we only have two subintervals. Thus,

$M_2 = h(3) \cdot \Delta x + h(7) \cdot \Delta x$
$= (2 + 6) \cdot \Delta x = 32$. 

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