Math 106B - Quiz 2 - January 22, 2007

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

1. (10 pts.) Let \( I = \int_1^7 \frac{1}{1+x} \, dx. \)

   (a) By hand, find \( L_3. \)

   \[
   \Delta x = \frac{b-a}{n} = \frac{7-1}{3} = \frac{6}{3} = 2. \quad \text{Sample points: } 1, 3, 5.
   \]

   \[
   L_3 = f(1) \Delta x + f(3) \Delta x + f(5) \Delta x.
   \]

   \[
   = \frac{1}{1+1} + \frac{1}{1+3} + \frac{1}{1+5}
   = \frac{1}{2} + \frac{1}{4} + \frac{1}{6}
   = 1 + \frac{1}{2} + \frac{1}{3} = \sqrt[6]{11}
   \]

   (b) Will \( L_3 \) be an overestimate or an underestimate of \( I \)? Explain. (A picture or a derivative may be helpful.)

   \[
   y = \frac{1}{1 + x}
   \]

   A left endpt sum will overestimate \( I \) because the function is decreasing.

   (Also, note that if \( f(x) = \frac{1}{1+x} = (1+x)^{-1} \),

   then \( f'(x) = -1(1+x)^{-2} = \frac{-1}{(1+x)^2} \).

   \( f'(x) \) is negative on \([1,7]\), so \( y = f(x) \) is a decreasing graph... so \( L_3 \) is an overestimate.)
2. (10 pts.) The graph of \( y = f'''(x) \) (NOT \( f(x) \)) is given above. Consider the integral \( I = \int_0^2 f(x) \, dx \).

Using a midpoint sum, how many intervals are necessary to guarantee accuracy to within \( \frac{1}{100} \) of \( I \)?

Need \( k_2 \) so that \( |f'''(x)| \leq k_2 \) for all \( x \) in \([0, 2]\).

By graph of \( y = f'''(x) \), we see that \( |f'''(x)| \leq 3 \).

let \( k_2 = 3 \).

\[ |I - M_n| \leq \frac{k_2(2-a)^3}{24n^2} \text{. We want this max error}
\]
\[ \text{to be } \leq \frac{1}{100} \text{.}
\]

\[ \frac{k_2(2-a)^3}{24n^2} \leq \frac{1}{100} \text{, so}
\]
\[ 3 \frac{(2-0)^3}{24n^2} \leq \frac{1}{100} \text{, so}
\]
\[ \frac{3 \cdot 8}{24n^2} \leq \frac{1}{100} \text{, so}
\]
\[ 100 \leq n^2, \text{ so } 10 \leq n \text{.}
\]

Thus, \( \frac{1}{100} \) at least \( 10 \) intervals are necessary.