Math 106 - Quiz 2 - January 18, 2008

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

For all questions on this quiz, let \( I = \int_0^3 x^2 \, dx \). Each question is worth 4 points.

1. By hand, evaluate \( R_3 \).
   \[
   \Delta x = \frac{b-a}{n} = \frac{3-0}{3} = 1.
   \]
   \[
   R_3 = f(1) \Delta x + f(2) \Delta x + f(3) \Delta x.
   \]
   \[
   = 1^2 \cdot 1 + 2^2 \cdot 1 + 3^2 \cdot 1 = 14
   \]

2. Using a theorem from class, what is the maximum possible error in using \( R_3 \) to approximate this integral \( I \)?
   \( R_3 \) needs \( K_1 \), needs \( f'(x) \). \( f(x) = x^2 \), so \( f'(x) = 2x \), and \( |2x| \) has max value \( 6 \) on \( [0, 3] \). So \( K_1 = 6 \).

   \[
   \left| I - R_3 \right| \leq K_1 \frac{(b-a)^2}{2n} = 6 \frac{(3-0)^2}{2(3)} = 9.
   \]

3. Use the FTC to evaluate \( I \) exactly.
   \[
   \int_0^3 x^2 \, dx = \frac{x^3}{3} \bigg|_0^3 = \frac{3^3}{3} - \frac{0^3}{3} = 9
   \]

4. What is the actual error in using \( R_3 \) to approximate \( I \)?
   \( I = 9 \), \( R_3 = 14 \).
   \[
   \left| I - R_3 \right| = 19 - 14 = 5
   \]

5. Using a theorem from class, how large should \( n \) be chosen to guarantee that \( |I - M_n| \leq \frac{1}{100} \)?
   \( M_n \) needs \( K_2 \), need \( f''(x) \). \( f(x) = x^2 \), \( f'(x) = 2x \), \( f''(x) = 2 \).
   Max value of \( |f''(x)| \) is \( 2 \). \( K_2 = 2 \).

   \[
   \text{Wont } n \text{ so that } |I - M_n| \leq K_2 \frac{(b-a)^3}{24\,n^2} \leq \frac{1}{100}.
   \]
   \[
   \frac{2(3-0)^3}{24\,n^2} \leq \frac{1}{100}, \text{ so } \frac{1400}{24} \leq n^2, \text{ so } 15 \leq n.
   \]
   \[
   n = 15 \text{ works!}
   \]