NAME:

Show ALL your work CAREFULLY.

For each of the following, use the method of substitution to find the indefinite integral. Be sure to indicate the substitution you use.

(a) \[ \int x^2 \sin(2 + x^3) \, dx \]

Let \( u = 2 + x^3 \) so that \( du = 3x^2 \, dx \). It follows that \( x^2 \, dx = \frac{1}{3} du \). We have

\[ \int x^2 \sin(2 + x^3) \, dx = \int \sin(2 + x^3) x^2 \, dx \]

and by substitution,

\[ \int \sin u \cdot \frac{1}{3} \, du = \frac{1}{3} \int \sin u \, du \]

\[ = \frac{1}{3} (-\cos u) + C = -\frac{1}{3} \cos(2 + x^3) + C. \]

(b) \[ \int \frac{x^3}{1 + x^2} \, dx \]

Let \( u = 1 + x^2 \) so that \( du = 2x \, dx \) or \( x \, dx = \frac{1}{2} du \). Now,

\[ \int \frac{x^3}{1 + x^2} \, dx = \int \frac{1}{1 + x^2} \cdot x^2 \cdot x \, dx \]

and by substitution,

\[ = \int \frac{1}{u} \cdot (u - 1) \cdot \frac{1}{2} \, du \]

\[ = \frac{1}{2} \int \frac{u - 1}{u} \, du = \frac{1}{2} \int 1 - \frac{1}{u} \, du \]

\[ = \frac{1}{2} (u - \ln|u|) + C = \frac{1}{2} (1 + x^2) - \ln(1 + x^2) + C. \]

Note that \( 1 + x^2 \) is always positive so \( \ln|1 + x^2| = \ln(1 + x^2) \).