MATH 106A,B - CALCULUS II  
WINTER 2009  
QUIZ 1

NAME:

Show ALL your work CAREFULLY.

(a) Use the method of substitution to find the following indefinite integral. Be sure to indicate the substitution you use.

\[
\int \frac{x^3}{1 + x^2} \, dx
\]

Let \( u = 1 + x^2 \). Then \( du = 2x \, dx \). The numerator of the integrand can be written as \( x^3 \, dx = x^2 x \, dx = (u - 1) \frac{1}{2} \, du \). It follows that

\[
\int \frac{x^3}{1 + x^2} \, dx = \int \frac{u - 1}{2} \, du = \frac{1}{2} \int \frac{1}{u} \, du = \frac{1}{2} (u - \ln |u|) + C = \frac{1}{2} (1 + x^2 - \ln(1 + x^2)) + C.
\]

(b) Use the method of substitution to evaluate the following definite integral. Be sure to indicate the substitution you use.

\[
\int_{\pi/2}^{\pi/2} \sin x \cos x \, dx
\]

Using the substitution \( u = \sin x \), we have \( du = \cos x \, dx \). Moreover, when \( x = 0, u = 0 \) and when \( x = \pi/2, u = 1 \). Thus,

\[
\int_{\pi/2}^{\pi/2} \sin x \cos x \, dx = \int_{0}^{1} u \, du = \left[ \frac{u^2}{2} \right]_{0}^{1} = \frac{1}{2}.
\]

Alternatively, one observes that \( \sin 2x = 2 \sin x \cos x \) thus

\[
\int_{0}^{\pi/2} \sin x \cos x \, dx = \frac{1}{2} \int_{0}^{\pi/2} \sin 2x \, dx = \frac{1}{2} \left[ \frac{-\cos 2x}{4} \right]_{0}^{\pi/2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.
\]

\[\text{Date: January 16, 2009.}\]