NAME:

Show ALL your work CAREFULLY.

(a) Use the method of substitution to evaluate the following definite integral. Be sure to indicate the substitution you use.

\[ \int_0^{\pi/2} \frac{\cos x}{1 + \sin x} \, dx. \]

Let \( u = 1 + \sin x \). It follows that \( du = \cos x \, dx \). When \( x = 0, u = 1 \) and when \( x = \pi/2, u = 2 \). Now our definite integral becomes

\[ \int_0^{\pi/2} \frac{\cos x}{1 + \sin x} \, dx = \int_1^2 \frac{du}{u} = \ln |u| \bigg|_1^2 = \ln 2 - \ln 1 = \ln 2. \]

(b) The graph of the function \( f \) is given below. Consider the area function \( F(x) = \int_0^x f(t) \, dt \). For what values of \( x \) on the interval \([0, 2]\) is \( F \) concave down?

The function \( F \) is concave down when the second derivative \( F'' \) is negative. By the Fundamental Theorem of Calculus, \( F'(x) = f(x) \) so that \( F''(x) = f'(x) \). Since \( f'(x) < 0 \) when \( f \) is decreasing, \( F \) is concave down when \( f \) is decreasing. Over the interval \([0, 2]\), \( f \) is decreasing for \( 1 < x \leq 2 \) (approximately).

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